THE DISTRIBUTION OF THE AVERAGE-RANGE FOR SUBGROUPS OF FIVE

BY

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1. Introduction.

The range in random samples from a normal population has come into widespread use as an estimator of the population standard deviation σ, and hence as a substitute for the sample standard deviation in several common statistical tests. However, the efficiency of the range as an estimator of σ decreases as the sample size N increases. For this reason it is often preferable to randomly divide the sample into a number of equal-sized subgroups and to find the mean of the several group ranges. This statistic, the sample average-range, will be denoted by $\bar{R}_{m,n}$, the subscript m referring to the number of subgroups and the subscript n to the subgroup size. The total sample size is $N = nm$.

The question as to which subdivision will be best from the point of view of smallest variance has been studied by Pearson [1] and Grubbs and Weaver [2]. It has been found that subgroups of seven or eight are best. However, in this report only sample sizes which are multiples of five are considered, i.e., samples of total size $N = 5m$. The average-range statistic will then be denoted by $\bar{R}_{m,5}$. One of the reasons for this choice of subgroup size is the fact that sample sizes which are multiples of five are in common use in acceptance sampling by variables, and the construction of the principal tables in this report was undertaken for the purpose of facilitating the devising of variables sampling plans based on the average-range. Reference to [2] bears out that very little efficiency is sacrificed by using subgroups of five observations instead of seven or eight.

The exact distribution of the average-range cannot in general be obtained in simple form. For this reason several approximations have been proposed. In particular, Patnaik [3] has suggested an approximation to the distribution of $\bar{R}_{m,n}$, based on equating its first two moments to the first two moments of a chi-variate. Because this approximation may be used in the construction of variables sampling plans it will be examined for accuracy by comparison with the exact distribution of the average-range, for which tables are given in this report. Also tabled herein are selected percentage points of the average-range statistic.

The ratio of a normal variate to range has been suggested as an analogue to Student's t-statistic, with similar uses in significance tests for population means, by Daly [4]. It is also pointed out by Patnaik that an approximation to the distribution of this ratio, where the denominator is the average-range instead of only a single range, is easily derived from his chi-approximation. This statistic was earlier considered by Lord [5] who studied it numerically. In this paper a table of exact percentage points are given for this ratio of a normal variate to the average range, when the range subgroups consist of five observations each.

2. <u>The Distribution of the Average-Range, $\bar{R}_{m,5}/\sigma$.</u>

Let $x_1, x_2, \ldots, x_n$ be a random sample of n observations, arranged in ascending order of magnitude, drawn from a normal population with arbitrary mean and variance σ^2. By the range in this sample is meant the difference between the largest and the smallest observations. If there are m such samples with n observations in each, the mean of the m ranges will be called the sample average-range or simply average-range, and will be denoted by $\bar{R}_{m,n}$. In particular, for $n = 5$ the exact distributions of $\bar{R}_{m,5}/\sigma$ are given in Table I, for $m = 1,2,\ldots,10$.

The entries in Table I were computed by successive convolutions on the numerical probability density function of a single range of five observations. This latter distribution had previously been obtained to eight places of accuracy by numerical integration. The results of these convolutions are the functions f_m of Table I. These were then summed to obtain the probability integrals F_m. The original eight decimal places were retained throughout the computations, rounding to the four places shown being done only for the final tabulation into Table I.

The accuracy of these tables was checked in the following ways, in addition to differencing: the means and variances were computed for each value of m by the formulas $\mu_m = \sum_i x_i f_m(x_i)$ and $\sigma_m^2 = \sum_i x_i^2 f(x_i) - \mu_m^2$, i.e., by numerical integration using the eight-decimal values of the probability densities f_m. These were found to agree to the known true values of the means and variances to at least five decimal places in each case.

Table II presents percentage points of the distribution of $\bar{R}_{m,5}/\sigma$. These were computed by inverse interpolation on the values of the probability integrals F_m. These percentage points may be used for significance tests for σ in the same ways in which tables of the percentage points of a chi-variate are used for tests for σ. Since Table II

TABLE I

TABLES OF THE DISTRIBUTION OF THE AVERAGE RANGE $\bar{R}_{5,m}$ OF SUBGROUPS OF FIVE ELEMENTS FROM A NORMAL POPULATION, IN TERMS OF THE POPULATION STANDARD DEVIATION.[1/]

$\bar{R}/\sigma$ \ m	f_1	F_1	f_2	F_2	f_3	F_3	f_4	F_4	f_5	F_5	m \ $\bar{R}/\sigma$
0	0	0	0	0	0	0	0	0	0	0	0
.05	0										.05
.1	.0002										.1
.15	.0008	.0000									.15
.20	.0018	.0001									.20
.25	.0035	.0002									.25
.30	.0059	.0004	.0000								.30
.35	.0093	.0008	.0001								.35
.40	.0137	.0014	.0001								.40
.45	.0192	.0022	.0003								.45
.50	.0260	.0033	.0006	.0000							.50
.55	.0339	.0048	.0011	.0001	.0000						.55
.60	.0432	.0068	.0020	.0002	.0001						.60
.65	.0537	.0092	.0033	.0003	.0002						.65
.70	.0655	.0121	.0053	.0005	.0004	.0000	.0000				.70
.75	.0785	.0157	.0081	.0008	.0007	.0001	.0001				.75
.80	.0928	.0200	.0120	.0013	.0013	.0001	.0001				.80
.85	.1081	.0250	.0172	.0021	.0024	.0002	.0003		.0000		.85
.90	.1245	.0309	.0241	.0031	.0040	.0004	.0006	.0000	.0001		.90
.95	.1418	.0375	.0327	.0045	.0065	.0006	.0012	.0001	.0002		.95
1.00	.1599	.0450	.0435	.0064	.0102	.0010	.0022	.0002	.0005	.0000	1.00
1.05	.1787	.0535	.0567	.0089	.0154	.0017	.0039	.0003	.0010	.0001	1.05
1.10	.1979	.0629	.0724	.0121	.0227	.0026	.0067	.0006	.0019	.0001	1.10
1.15	.2175	.0733	.0908	.0162	.0324	.0040	.0109	.0010	.0035	.0003	1.15
1.20	.2373	.0847	.1119	.0212	.0452	.0059	.0172	.0017	.0063	.0005	1.20
1.25	.2572	.0970	.1358	.0274	.0614	.0086	.0261	.0028	.0108	.0009	1.25
1.30	.2769	.1104	.1625	.0349	.0817	.0122	.0386	.0044	.0176	.0017	1.30
1.35	.2962	.1247	.1917	.0437	.1062	.0169	.0553	.0068	.0279	.0028	1.35
1.40	.3151	.1400	.2232	.0541	.1353	.0229	.0772	.0101	.0426	.0046	1.40
1.45	.3334	.1562	.2566	.0661	.1692	.0305	.1048	.0146	.0629	.0072	1.45
1.50	.3509	.1733	.2917	.0798	.2076	.0399	.1390	.0207	.0900	.0110	1.50
1.55	.3675	.1913	.3279	.0953	.2505	.0514	.1800	.0287	.1251	.0164	1.55
1.60	.3830	.2101	.3646	.1126	.2972	.0651	.2279	.0389	.1690	.0237	1.60
1.65	.3974	.2296	.4014	.1317	.3472	.0812	.2824	.0517	.2222	.0335	1.65
1.70	.4105	.2498	.4376	.1527	.3995	.0999	.3429	.0673	.2848	.0462	1.70

[1/] The random variable $\bar{R}_{5,m}$, is the average of the m ranges of m independent samples of size 5 drawn from a normal population with arbitrary mean and variance σ^2. The distribution of $\bar{R}$ is independent of μ. The tables given above, labeled f_m and F_m, are the probability densities and probability integrals, respectively, for m = 1,2,...,10, of $\bar{R}/\sigma$.

For m > 10 the distribution is nearly normal with mean 2.32593 and variance $\frac{.74664}{m}$.

m / $\bar{R}/\sigma$	f_6	F_6	f_7	F_7	f_8	F_8	f_9	F_9	f_{10}	F_{10}	m / $\bar{R}/\sigma$
0	0	0	0	0	0	0	0	0	0	0	0
.05											.05
.10											.10
.15											.15
.20											.20
.25											.25
.30											.30
.35											.35
.40											.40
.45											.45
.50											.50
.55											.55
.60											.60
.65											.65
.70											.70
.75											.75
.80											.80
.85											.85
.90											.90
.95	.0000										.95
1.00	.0001		.0000								1.00
1.05	.0002		.0001								1.05
1.10	.0005	.0000	.0001		.0000						1.10
1.15	.0011	.0001	.0004		.0001		.0000				1.15
1.20	.0023	.0002	.0008	.0000	.0003		.0001		.0000		1.20
1.25	.0043	.0003	.0017	.0001	.0007	.0000	.0003		.0001		1.25
1.30	.0079	.0006	.0035	.0002	.0015	.0001	.0007	.0000	.0003		1.30
1.35	.0138	.0012	.0067	.0005	.0032	.0002	.0015	.0001	.0007	.0000	1.35
1.40	.0230	.0021	.0122	.0010	.0064	.0005	.0034	.0002	.0017	.0001	1.40
1.45	.0369	.0036	.0214	.0018	.0122	.0009	.0069	.0005	.0039	.0002	1.45
1.50	.0571	.0059	.0357	.0032	.0221	.0018	.0135	.0010	.0082	.0005	1.50
1.55	.0851	.0095	.0571	.0056	.0379	.0033	.0250	.0019	.0163	.0012	1.55
1.60	.1227	.0147	.0878	.0092	.0622	.0058	.0437	.0037	.0305	.0023	1.60
1.65	.1712	.0220	.1300	.0146	.0977	.0098	.0729	.0066	.0540	.0044	1.65
1.70	.2316	.0321	.1857	.0225	.1473	.0159	.1159	.0113	.0907	.0081	1.70
1.75	.3041	.0455	.2560	.0336	.2132	.0249	.1762	.0186	.1447	.0139	1.75
1.80	.3880	.0628	.3413	.0485	.2970	.0377	.2564	.0294	.2201	.0231	1.80
1.85	.4815	.0845	.4405	.0680	.3987	.0551	.3580	.0448	.3196	.0366	1.85
1.90	.5821	.1111	.5512	.0928	.5166	.0780	.4803	.0657	.4437	.0556	1.90
1.95	.6859	.1428	.6695	.1233	.6467	.1070	.6198	.0932	.5902	.0815	1.95
2.00	.7885	.1797	.7900	.1598	.7833	.1428	.7706	.1280	.7531	.1151	2.00
2.05	.8851	.2215	.9066	.2022	.9189	.1853	.9240	.1704	.9232	.1570	2.05
2.10	.9708	.2679	1.0125	.2502	1.0450	.2344	1.0699	.2202	1.0885	.2073	2.10
2.15	1.0414	.3182	1.1017	.3031	1.1533	.2894	1.1977	.2769	1.2362	.2654	2.15
2.20	1.0931	.3716	1.1686	.3598	1.2363	.3491	1.2976	.3393	1.3535	.3301	2.20

m / $\bar{R}/\sigma$	f_1	F_1	f_2	F_2	f_3	F_3	f_4	F_4	f_5	F_5	m / $\bar{R}/\sigma$
1.75	.4223	.2706	.4726	.1755	.4530	.1212	.4084	.0861	.3561	.0622	1.75
1.80	.4326	.2920	.5058	.1999	.5066	.1452	.4772	.1082	.4348	.0820	1.80
1.85	.4415	.3138	.5367	.2260	.5591	.1718	.5476	.1338	.5189	.1058	1.85
1.90	.4488	.3361	.5648	.2536	.6090	.2010	.6176	.1630	.6059	.1339	1.90
1.95	.4546	.3587	.5896	.2824	.6553	.2326	.6850	.1955	.6926	.1664	1.95
2.00	.4589	.3816	.6107	.3124	.6966	.2664	.7474	.2313	.7757	.2031	2.00
2.05	.4616	.4046	.6278	.3434	.7320	.3021	.8029	.2701	.8518	.2438	2.05
2.10	.4627	.4277	.6407	.3751	.7607	.3395	.8494	.3114	.9176	.2880	2.10
2.15	.4624	.4508	.6493	.4074	.7818	.3780	.8856	.3548	.9704	.3352	2.15
2.20	.4606	.4739	.6534	.4400	.7951	.4174	.9102	.3997	1.0079	.3847	2.20
2.25	.4574	.4969	.6532	.4727	.8004	.4573	.9225	.4455	1.0287	.4356	2.25
2.30	.4528	.5196	.6488	.5052	.7976	.4973	.9226	.4916	1.0323	.4871	2.30
2.35	.4470	.5421	.6403	.5375	.7872	.5369	.9105	.5375	1.0189	.5384	2.35
2.40	.4400	.5643	.6281	.5692	.7696	.5758	.8872	.5824	.9896	.5886	2.40
2.45	.4318	.5861	.6123	.6002	.7454	.6137	.8538	.6259	.9461	.6370	2.45
2.50	.4227	.6075	.5935	.6304	.7156	.6502	.8117	.6676	.8909	.6829	2.50
2.55	.4126	.6283	.5719	.6595	.6808	.6851	.7626	.7069	.8264	.7259	2.55
2.60	.4017	.6487	.5481	.6875	.6422	.7182	.7082	.7437	.7555	.7654	2.60
2.65	.3901	.6685	.5223	.7143	.6007	.7493	.6502	.7777	.6809	.8013	2.65
2.70	.3779	.6877	.4951	.7397	.5573	.7782	.5904	.8087	.6051	.8335	2.70
2.75	.3651	.7063	.4668	.7638	.5129	.8050	.5303	.8367	.5305	.8619	2.75
2.80	.3520	.7242	.4379	.7864	.4682	.8295	.4712	.8617	.4588	.8866	2.80
2.85	.3384	.7415	.4086	.8076	.4242	.8518	.4144	.8839	.3917	.9079	2.85
2.90	.3247	.7580	.3795	.8273	.3814	.8720	.3607	.9032	.3301	.9259	2.90
2.95	.3107	.7739	.3507	.8455	.3403	.8900	.3108	.9200	.2747	.9410	2.95
3.00	.2967	.7891	.3225	.8623	.3014	.9061	.2652	.9344	.2256	.9536	3.00
3.05	.2827	.8036	.2952	.8778	.2651	.9202	.2241	.9467	.1833	.9638	3.05
3.10	.2687	.8174	.2689	.8919	.2315	.9326	.1876	.9570	.1471	.9720	3.10
3.15	.2549	.8305	.2439	.9047	.2007	.9434	.1555	.9655	.1166	.9786	3.15
3.20	.2413	.8429	.2202	.9163	.1729	.9528	.1278	.9726	.0914	.9838	3.20
3.25	.2279	.8546	.1979	.9267	.1479	.9608	.1040	.9784	.0708	.9879	3.25
3.30	.2148	.8657	.1771	.9361	.1257	.9676	.0839	.9831	.0543	.9910	3.30
3.35	.2020	.8761	.1578	.9445	.1061	.9734	.0671	.9869	.0411	.9934	3.35
3.40	.1896	.8859	.1400	.9519	.0890	.9783	.0532	.9899	.0308	.9952	3.40
3.45	.1777	.8951	.1237	.9585	.0741	.9824	.0418	.9923	.0229	.9965	3.45
3.50	.1661	.9037	.1088	.9643	.0614	.9858	.0326	.9941	.0168	.9975	3.50
3.55	.1550	.9117	.0954	.9694	.0505	.9886	.0252	.9956	.0122	.9983	3.55
3.60	.1444	.9192	.0832	.9739	.0413	.9909	.0193	.9967	.0087	.9988	3.60
3.65	.1342	.9261	.0724	.9778	.0336	.9927	.0147	.9975	.0062	.9992	3.65
3.70	.1245	.9326	.0627	.9811	.0272	.9943	.0111	.9982	.0044	.9994	3.70
3.75	.1153	.9386	.0541	.9840	.0218	.9955	.0083	.9987	.0031	.9996	3.75
3.80	.1066	.9441	.0464	.9865	.0174	.9965	.0062	.9990	.0021	.9997	3.80
3.85	.0983	.9493	.0398	.9887	.0139	.9973	.0045	.9993	.0014	.9998	3.85
3.90	.0906	.9540	.0339	.9905	.0109	.9979	.0033	.9995	.0010	.9999	3.90
3.95	.0832	.9583	.0288	.9921	.0086	.9984	.0024	.9996	.0007	.9999	3.95

F/σ \ m	f_6	F_6	f_7	F_7	f_8	F_8	f_9	F_9	f_{10}	F_{10}	m / F/σ
2.25	1.1234	.4270	1.2094	.4193	1.2884	.4123	1.3617	.4058	1.4303	.3997	2.25
2.30	1.1312	.4834	1.2220	.4801	1.3063	.4771	1.3854	.4744	1.4602	.4720	2.30
2.35	1.1166	.5396	1.2063	.5408	1.2896	.5420	1.3678	.5433	1.4416	.5445	2.35
2.40	1.0809	.5945	1.1640	.6000	1.2404	.6053	1.3113	.6103	1.3776	.6150	2.40
2.45	1.0269	.6472	1.0986	.6566	1.1632	.6654	1.2218	.6736	1.2752	.6813	2.45
2.50	.9576	.6968	1.0148	.7094	1.0641	.7210	1.1070	.7318	1.1444	.7418	2.50
2.55	.8771	.7427	.9178	.7577	.9503	.7714	.9761	.7839	.9964	.7953	2.55
2.60	.7894	.7843	.8131	.8010	.8288	.8159	.8382	.8292	.8423	.8413	2.60
2.65	.6983	.8215	.7061	.8390	.7064	.8543	.7012	.8677	.6917	.8797	2.65
2.70	.6074	.8542	.6011	.8717	.5887	.8866	.5720	.8996	.5522	.9108	2.70
2.75	.5197	.8824	.5020	.8993	.4799	.9134	.4551	.9252	.4288	.9353	2.75
2.80	.4376	.9063	.4114	.9221	.3828	.9349	.3533	.9454	.3241	.9541	2.80
2.85	.3626	.9263	.3310	.9407	.2990	.9520	.2679	.9610	.2385	.9682	2.85
2.90	.2959	.9428	.2615	.9555	.2287	.9652	.1984	.9726	.1710	.9784	2.90
2.95	.2378	.9561	.2029	.9671	.1714	.9752	.1436	.9812	.1196	.9857	2.95
3.00	.1883	.9668	.1548	.9760	.1259	.9826	.1016	.9873	.0815	.9907	3.00
3.05	.1469	.9751	.1160	.9828	.0907	.9881	.0703	.9916	.0542	.9941	3.05
3.10	.1130	.9816	.0855	.9878	.0641	.9919	.0476	.9946	.0352	.9963	3.10
3.15	.0856	.9866	.0620	.9915	.0444	.9946	.0316	.9965	.0223	.9978	3.15
3.20	.0640	.9903	.0442	.9942	.0302	.9965	.0205	.9978	.0138	.9987	3.20
3.25	.0472	.9931	.0310	.9961	.0202	.9977	.0130	.9987	.0084	.9992	3.25
3.30	.0344	.9952	.0214	.9974	.0132	.9986	.0081	.9992	.0049	.9996	3.30
3.35	.0247	.9966	.0146	.9983	.0085	.9991	.0050	.9995	.0029	.9998	3.35
3.40	.0175	.9977	.0098	.9989	.0054	.9994	.0030	.9997	.0016	.9999	3.40
3.45	.0122	.9984	.0064	.9993	.0034	.9997	.0017	.9998	.0009	.9999	3.45
3.50	.0084	.9989	.0042	.9995	.0021	.9998	.0010	.9999	.0005	1.0000	3.50
3.55	.0058	.9993	.0027	.9997	.0012	.9999	.0006	1.0000	.0003		3.55
3.60	.0039	.9995	.0017	.9998	.0007	.9999	.0003		.0001		3.60
3.65	.0026	.9997	.0011	.9999	.0004	1.0000	.0002		.0001		3.65
3.70	.0017	.9998	.0006	.9999	.0002		.0001		.0000		3.70
3.75	.0011	.9999	.0004	1.0000	.0001		.0000				3.75
3.80	.0007	.9999	.0002		.0001						3.80
3.85	.0004	1.0000	.0001		.0000						3.85
3.90	.0003		.0001								3.90
3.95	.0002		.0000								3.95
4.00	.0001										4.00
4.05	.0001										4.05
4.10	.0000										4.10
4.45											4.45

R/σ \ m	f_1	F_1	f_2	F_2	f_3	F_3	f_4	F_4	f_5	F_5	m \ R/σ
4.00	.0764	.9623	.0244	.9934	.0067	.9987	.0017	.9998	.0004	.9999	4.00
4.05	.0700	.9660	.0206	.9946	.0052	.9990	.0012	.9998	.0003	1.0000	4.05
4.10	.0640	.9693	.0173	.9955	.0040	.9993	.0009	.9999	.0002		4.10
4.15	.0584	.9724	.0145	.9963	.0031	.9995	.0006	.9999	.0001		4.15
4.20	.0532	.9752	.0121	.9970	.0024	.9996	.0004	.9999	.0001		
4.25	.0484	.9777	.0100	.9975	.0018	.9997	.0003	1.0000	.0000		4.25
4.30	.0440	.9800	.0083	.9980	.0014	.9998	.0002				4.30
4.35	.0399	.9821	.0069	.9983	.0010	.9998	.0001				4.35
4.40	.0361	.9840	.0057	.9987	.0008	.9999	.0001				4.40
4.45	.0326	.9857	.0046	.9989	.0006	.9999	.0001				4.45
4.50	.0295	.9873	.0038	.9991	.0004	.9999	.0000				4.50
4.55	.0265	.9887	.0031	.9993	.0003	1.0000					4.55
4.60	.0239	.9899	.0025	.9994	.0002						4.60
4.65	.0214	.9911	.0020	.9995	.0002						4.65
4.70	.0192	.9921	.0016	.9996	.0001						4.70
4.75	.0172	.9930	.0013	.9997	.0001						4.75
4.80	.0154	.9938	.0011	.9998	.0001						4.80
4.85	.0137	.9945	.0008	.9998	.0000						4.85
4.90	.0122	.9952	.0007	.9999							4.90
4.95	.0109	.9958	.0005	.9999							4.95
5.00	.0097	.9963	.0004	.9999							5.00
5.05	.0086	.9967	.0003	.9999							5.05
5.10	.0076	.9971	.0003	.9999							5.10
5.15	.0067	.9975	.0002	1.0000							5.15
5.20	.0059	.9978	.0002								5.20
5.25	.0052	.9981	.0001								5.25
5.30	.0046	.9983	.0001								5.30
5.35	.0040	.9985	.0001								5.35
5.40	.0036	.9987	.0001								5.40
5.45	.0031	.9989	.0000								5.45
5.50	.0027	.9991									5.50
5.55	.0024	.9992									5.55
5.60	.0021	.9993									5.60
5.65	.0018	.9994									5.65
5.70	.0016	.9995									5.70
5.75	.0014	.9995									5.75
5.80	.0012	.9996									5.80
5.85	.0010	.9997									5.85
5.90	.0009	.9997									5.90
5.95	.0008	.9998									5.95
6.00	.0007	.9998									6.00
6.05	.0006	.9998									6.05
6.10	.0005	.9998									6.10
6.15	.0004	.9999									6.15
6.20	.0004	.9999									6.20

m / $\bar{R}/\sigma$	f_1	F_1	f_2	F_2	f_3	F_3	f_4	F_4	f_5	F_5	m / $\bar{R}/\sigma$
6.25	.0003	.9999	0	1.0000	0	1.0000	0	1.0000	0	1.0000	6.25
6.30	.0003	.9999									6.30
6.35	.0002	.9999									6.35
6.40	.0002	.9999									6.40
6.45	.0002	1.0000									6.45
6.50	.0001										6.50
6.55	.0001										6.55
6.60	.0001										6.60
6.65	.0001										6.65
6.70	.0001										6.70
6.75	.0001										6.75
6.80	.0001										6.80
	0										
7.05	0										7.05

gives the percentage points $\bar{R}_p$ of $\bar{R}_{m,5}/\sigma$ then $Pr\left\{\bar{R}_{m,5} \geq \bar{R}_{p}\sigma\right\} = P$. For example, $Pr\left\{\bar{R}_{4,5} \geq 1.528\sigma\right\} = .975$ for four groups of five, i.e., for samples of size twenty.

Confidence limits for σ can be obtained from Table II since

$$Pr\left\{\frac{\bar{R}_{m,5}}{\bar{R}_{p_2}} \leq \sigma \leq \frac{\bar{R}_{m,5}}{\bar{R}_{p_1}}\right\} = P_2 - P_1.$$ As an example, for $m = 5$, Table II gives

$$Pr\left\{\frac{\bar{R}_{5,5}}{3.394} \leq \sigma \leq \frac{\bar{R}_{5,5}}{1.410}\right\} = .995 - .005 = .99.$$

For tests of hypotheses concerning σ, suppose a preassigned value is σ_o and the alternatives are $\sigma > \sigma_o$, we find as a critical region at the α level of significance $Pr\left\{\bar{R}_{m,5} > \sigma_o\bar{R}_\alpha \mid \sigma = \sigma_o\right\} = Pr\left\{\bar{R}_{m,5}/\sigma_o > \bar{R}_\alpha\right\} = \alpha$. For example, if $m = 2$ and $\alpha = .05$ a critical region would be $\bar{R}_{2,5} > 3.387\sigma_o$, i.e., one would reject the hypothesis that $\sigma = \sigma_o$ at the .05 significance level if the observed value of the average-range exceeded $3.387\sigma_o$.

3. <u>The Average-Range Analogue to the t-Test</u>.

Let z be distributed normally with mean zero and standard deviation one, and let $\bar{R}_{m,n}$ be an independent average-range statistic. Consider the statistic $t_{m,n} = \frac{z}{\bar{R}_{mn}}$. The statistic $d_n t_{m,n}$ [1] was studied by Lord [5] who gave some percentage points and applications. Table III contained herein gives all the commonly used percentage points for $t_{m,5}$ for $m = 2, 3, \ldots, 10$. The values given in Table III were obtained by numerical quadrature using the normal probability integral and the probability densities of Table I.

Since the average-range of a sample from a normal population is independent of the mean of that sample the statistic $\frac{\sqrt{m.n}\,(\bar{x} - \mu)}{\bar{R}_{m,n}}$ is

---

[1] d_n is the expected value of R_n/σ where R_n is a single range of size n.

TABLE II

Percentage Points of the Average-Range Statistic, $\bar{R}_{m,5}/\sigma$

$$P = \Pr\left\{\bar{R}_{m,5}/\sigma \geq \bar{R}_0\right\}$$

$m \backslash P$ ($\bar{R}_o$)	.005	.010	.025	.050	.100	.250	.750	.900	.950	.975	.99	.995
2	4.073	3.885	3.614	3.387	3.131	2.721	1.893	1.564	1.381	1.231	1.069	.965
3	3.729	3.580	3.365	3.184	2.981	2.651	1.976	1.700	1.544	1.415	1.272	1.178
4	3.529	3.402	3.219	3.065	2.891	2.609	2.025	1.782	1.644	1.528	1.399	1.314
5	3.394	3.282	3.121	2.985	2.831	2.580	2.057	1.839	1.713	1.607	1.489	1.410
6	3.296	3.195	3.049	2.926	2.786	2.558	2.081	1.880	1.765	1.666	1.556	1.483
7	3.220	3.128	2.994	2.880	2.751	2.542	2.100	1.913	1.804	1.713	1.609	1.540
8	3.159	3.074	2.949	2.844	2.724	2.528	2.115	1.939	1.837	1.750	1.652	1.587
9	3.110	3.029	2.913	2.813	2.701	2.517	2.127	1.961	1.864	1.782	1.689	1.626
10	3.068	2.993	2.881	2.788	2.681	2.507	2.137	1.979	1.887	1.808	1.720	1.660

distributed as the random variable for which percentage points are given in Table III. Thus we can test for the significance of the difference between the mean $\bar{x}$ of the sample and a preassigned value μ, in exactly the same way that $\frac{\sqrt{N}(\bar{x}-\mu)}{s}$ is used, where s is the sample standard deviation $\sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}}$.

Suppose a test hypothesis is that $\mu = \mu_o$, and the alternative hypothesis is that $\mu > \mu_o$. A critical region at the α level of significance becomes $\frac{\sqrt{5m}(\bar{x}-\mu_o)}{\bar{R}_{m,5}} > t_\alpha$ where t_α is the α percentage point in the row labeled m in Table III. In particular, if $m = 4$, $\alpha = .01$, one would reject the hypothesis that $\mu = \mu_o$ whenever

$$\frac{\sqrt{20}(\bar{x}-\mu_o)}{\bar{R}_{4,5}} > 1.101$$

Table III may be used in any other test of significance for which one would use a table of the percentage points of Student's t-statistic, such as confidence intervals for the population mean, comparison of two means, and so on.

4. The χ-Approximation to the Distribution of $\bar{R}_{m,n}$.

It has been proposed by Patnaik [3] that $\bar{R}_{m,n}/\sigma$ is approximately distributed as $c\chi_\nu/\sqrt{\nu}$ where χ_ν is a chi-variate with ν degrees of freedom, and both ν and the scale factor c depend on m and n. These constants $c_{m,n}$ and $\nu_{m,n}$ are obtained by equating the first two moments of $\bar{R}_{m,n}/\sigma$ to the first two moments of $\frac{s}{\sigma} = \sqrt{\frac{\sum(x_i-\bar{x})^2}{(n-1)\sigma^2}}$. The usefulness of this approximation in variables sampling inspection stems from the fact

TABLE III

Percentage Points of the Average-Range Analogue of Student's-t[1/]

$$P = Pr\left\{t_{m,5} \leq t_o\right\}$$

m \ P	.75	.90	.95	.975	.99	.995
	t_o					
2	.295	.584	.780	.969	1.221	1.427
3	.293	.573	.754	.923	1.139	1.299
4	.292	.567	.742	.902	1.101	1.246
5	.292	.564	.735	.890	1.080	1.217
6	.292	.562	.730	.881	1.066	1.198
7	.291	.560	.727	.876	1.056	1.184
8	.291	.559	.724	.871	1.049	1.174
9	.291	.558	.722	.868	1.043	1.166
10	.291	.557	.721	.866	1.039	1.160

[1/] The statistic $t_{m,n} = \frac{z}{\bar{R}_{m,n}}$ is the ratio of a standardized normal variate to an independent average-range statistic from a normal population with arbitrary mean and variance equal to one. The distribution is symmetric around zero so $t_o(1-P) = -t_o(P)$. For m large and any $n \geq 2$ the Cornish-Fisher expansion for $c_{mn}t$ where t is Student's-t with degrees of freedom replaced by ν_{mn} will provide a highly accurate method for obtaining these percentage points. The constants c_{mn} and ν_{mn} can be obtained from [3].

that since $\bar{R}_{m,n}$ and $\bar{x}$ are independent it follows that $\frac{\sqrt{mn}\, c_{mn}(U-\bar{x})}{\bar{R}_{m,n}}$ is distributed as non-central t, with parameters ν and $\sqrt{mn}\,U$. In a variables sampling plan the acceptance criterion is commonly written $\bar{x}+k\bar{R} \leq U$ where U is an upper limit defining quality. This can be written $\frac{U-\bar{x}}{\bar{R}} \geq k$, and, multiplying by the appropriate constants, as $\sqrt{mn}\, c_{mn} \frac{U-\bar{x}}{\bar{R}} \geq \sqrt{mn}\, c_{mn} k$. Therefore the probability that a lot with fraction defective p will be accepted by the plan is approximately equal to the probability that a non-central t variate (with degrees of freedom $\nu_{m,n}$ and eccentricity $\sqrt{mn}\, K_p$ [1/]) will exceed $\sqrt{mn}\, c_{mn} k = k'$.

Explicit instructions for the use of the non-central t tables [6] in variables sampling are given by Wallis in [7] and will not be repeated here. It is to be noted that the only essential modification to those instructions is the necessity of replacing the degrees of freedom f by the appropriate $\nu_{m,n}$. Values of $\nu_{m,n}$ can be obtained from Patnaik's original paper [3].

The author has compared the accuracy of the chi-approximation to $\bar{R}_{m,5}/\sigma$ with the values of Table I, and found it to be excellent for subgroups of five even for a small number of subgroups. A summary of this comparison is given in Table IV.

As pointed out by Patnaik, as a consequence of the chi-approximation, the statistic $\frac{c_{m,n}\sqrt{mn}(\bar{x}-\mu)}{\bar{R}}$ is distributed approximately as Student's t with $\nu_{m,n}$ degrees of freedom, and the statistic $c_{m,n}\sqrt{mn} \cdot \frac{\left(\frac{\bar{x}-\mu}{\sigma}\right) + \left(\frac{\mu-\mu_o}{\sigma}\right)}{\frac{\bar{R}}{\sigma}}$ is distributed approximately as non-central t, with $\nu_{m,n}$ degrees of

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1/ K_p is the standardized normal deviate exceeded with probability p, i.e., $\int_{K_p}^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} = p$.

TABLE IV

m = 2

$\bar{R}_{2,5}/\sigma$	4.073	2.721	1.893	.965
Exact prob.	.995	.750	.250	.005
χ-approximation	.996	.748	.249	.006

m = 5

$\bar{R}_{5,5}/\sigma$	3.394	2.580	2.057	1.410
Exact prob.	.995	.750	.250	.005
χ-approximation	.996	.749	.249	.005

m = 8

$\bar{R}_{8,5}/\sigma$	3.159	2.528	2.115	1.587
Exact prob.	.995	.750	.250	.005
χ-approximation	.995	.749	.249	.005

freedom and eccentricity $\frac{\sqrt{mn}\,(\mu - \mu_0)}{\sigma}$. The accuracy of these two approximations was also checked by numerical methods involving the exact values in Table I, and also found to be very good for the cases considered. For the purposes of computing operating characteristic curves for variables sampling plans or for finding the power of the t-test analogue of Section 2, the approximation gave results which were superior to those obtained by assuming that $\bar{x} + k\bar{R}_{m,5}$ is approximately normally distributed.

Since $\bar{R}_{m,n}$ is an average of identically distributed variables with finite first and second moments, for m large the average-range statistic is very nearly normal. For this reason it was not considered necessary to compute the distribution for values of m greater than ten, nor would the χ-approximation be an advantage over the normal for these larger values of m. For reference purposes it is noted that accurate values of the expected values and variances of the average-range statistic $\bar{R}_{m,5}/\sigma$ are:

$$E\left(\frac{\bar{R}_{m,5}}{\sigma}\right) = 2.32592895$$

$$Var\left(\frac{\bar{R}_{m,5}}{\sigma}\right) = \frac{.74663760}{m}$$

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